

## **Taiwan-Waseda joint workshop on partial differential equations and mathematical modelings**

Location: Building 55, Room 02-01, Nishi-Waseda Campus, Waseda University

Date: August 30, 2024

Organizers : Hideo Kozono, Kousuke Kuto, Ken-ichi Maruno (Waseda Univ.)

10am-10:50am      Ying-Chieh Lin (National University of Kaohsiung)  
Global entropy solution and relaxation limit for Greenberg-Klar-Rascle multi-lane  
traffic flow model

11am-11:30am      Jou-Chun Kuo (Waseda University)  
 $L_1$  approach to the compressible viscous fluid flows in the half-space

11:30pm-12pm      Tomoharu Kinoshita (Waseda University)  
Existence and multiplicity of radially symmetric solutions for nonlinear Schrödinger  
equations

Lunch time

1:30pm-2:20pm      Shih-Wei Chou (Soochow University)  
Global Transonic Solutions of the Hot Jupiter Model for Atmospheric Escape Behavior

2:30pm-3pm          Tatsuki Yamamoto (Waseda University)  
Nonhomogeneous boundary value problem for the steady Navier-Stokes equations in  
multiply-connected domains

3:20pm-4:10pm      Hsin-Yi Lee (National Chiayi University)  
Global Classical ( $C^1$ ) Solutions for Non-isentropic Gas in the Nozzle Flows

4:20pm-4:50pm      Yoshiki Iida (Waseda University)  
On the global well-posedness of the primitive equations on non-flat layers

4:50pm-5:20pm      Taichi Eguchi (Waseda University)  
The energy equality of the fractional Navier—Stokes equations in the framework of  
the Lorentz—Besov spaces

**Title:** Global entropy solution and relaxation limit for Greenberg-Klar-Rascle multi-lane traffic flow model

**Speaker:** Ying-Chieh Lin (National University of Kaohsiung)

**Abstract:** In this talk, we consider the Greenberg-Klar-Rascle multi-lane traffic flow model. This model is a relaxation system with the equilibrium state that is a discontinuous function of the car density. We study the existence of global entropy solutions and the relaxation limit for the GKR model. To construct the approximate solutions, we find two sequences of invariant regions under some suitable condition of initial data. As the relaxation time approaches 0, we prove that the limit of the entropy solutions for the GKR model is a weak solution of its equilibrium equation. It is interesting that the equilibrium equation is a scalar conservation law with discontinuous flux.

# $L_1$ approach to the compressible viscous fluid flows in the half-space

Jou-Chun Kuo

Waseda University

kuojouchun@asagi.waseda.jp

This talk is based on a joint work with Professor Emeritus Yoshihiro Shibata (Waseda University).

This talk is devoted to proving the local well-posedness for the Navier-Stokes equations describing the isotropic motion of the compressible viscous fluid flows in the half-space with non-slip boundary conditions. We construct a unique strong solution in the  $L_1$  in time and  $B_{q,1}^{s+1} \times B_{q,1}^s$  in space maximal regularity class. Here,  $1 < q < \infty$  and  $-1 + N/q \leq s < 1/q$ , where  $N \geq 2$  is the space dimension. The approach is by means of the analytic semigroup theory and we use Lagrange transformation in order to eliminate the convection term  $\mathbf{v} \cdot \nabla \rho$ .

# Existence and multiplicity of radially symmetric solutions for nonlinear Schrödinger equations

Tomoharu Kinoshita  
Waseda University

In this talk, we study the nonlinear Schrödinger equations with Berestycki-Lions type nonlinearities and show the existence and multiplicity of radially symmetric solutions by variational method. Azzollini-Pomponio(2010) and Liu-Liu-Liao(2019) considered the case that the potential has constant sign. In our case, we deal with the potential which is allowed to be sign changing.

**Title:** Global Transonic Solutions of the Hot Jupiter Model for Atmospheric Escape Behavior

**Speaker:** Shih-Wei Chou (Soochow University)

**Abstract:** In this talk, we present modifications to the original hot-Jupiter model, which addresses the problem of hydrodynamic escape in the planetary atmosphere. The model incorporates the Euler equation with gravity, tidal effects, and heat. We employ the generalized Glimm technique to prove the existence of transonic solutions to this problem. By adjusting the dilation of the characteristic fields, we enhance the accuracy of the Glimm–Goodman wave interaction estimates. This improvement allows us to establish a more general admissible condition for stabilizing the generalized Glimm scheme. Additionally, we derive the exact relationship between the lower bound of the gas velocity in the subsonic state and the adiabatic constant of the gas. Under certain constraints on the transonic initial and boundary data, the limit of the approximated solutions represents an entropy transonic solution with bounded variations.

# Nonhomogeneous boundary value problem for the steady Navier-Stokes equations in multiply-connected domains

Tatsuki Yamamoto  
Waseda University, Japan

We consider the nonhomogeneous boundary value problem for the steady Navier-Stokes equations under the slip boundary conditions in a two-dimensional bounded domain with multiple boundary components. By the incompressibility condition of the fluid, the total flux of the given boundary datum through the boundary must be zero. We prove that this problem has a solution if the friction coefficient is sufficiently large compared with the kinematic viscosity constant and the curvature of the boundary. No additional assumption (other than the necessary requirement of zero total flux through the boundary) is imposed on the boundary data. We also show that such an assumption on the friction coefficient is redundant for the existence of a solution in the case when the fluxes across each connected component of the boundary are sufficiently small, or the domain and the given data satisfy certain symmetry conditions. This talk is based on the joint work with Prof. Giovanni P. Galdi (University of Pittsburgh).

**Title:** Global Classical ( $C^1$ ) Solutions for Non-isentropic Gas in the Nozzle Flows

**Speaker:** Hsin-Yi Lee (National Chiayi University)

**Abstract:** In this talk, we investigate the global existence of classical solutions for non-isentropic gas flows through a duct. This problem can be described as an initial-boundary value problem for the full compressible Euler equations with the geometric source in Lagrangian coordinates, which can be viewed as a hyperbolic system of balance laws when the Riemann invariants are applied. We prove the global existence theorem for classical solutions under appropriate conditions on entropies, ducts, and initial and boundary data. Our analysis mainly depends on the local existence theorem and uniform a priori estimates, which can be obtained by using the method of characteristics and introducing generalized Lax transformations. Furthermore, the long-time behavior of global classical solutions along all characteristic curves and vertical lines is also determined.

This is joint work with Shih-Wei Chou, John M. Hong, and Shih-Ming Wang.

# On the global well-posedness of the primitive equations on non-flat layers

— Taiwan-Waseda joint workshop on partial differential equations and  
mathematical modelings —

Yoshiki Iida (Waseda university)

July 5, 2024

## Abstract

In this talk, we consider the primitive equations on non-flat layers. This system is deduced from the anisotropic Navier–Stokes equations via hydrostatic approximation. Firstly, we discuss how to define the hydrostatic Helmholtz projection and the hydrostatic Stokes operator without the flatness of the layers, and we then prove the maximal  $L^q$ -regularity of the corresponding linearized equation via  $H^\infty$ -calculus. By combining the maximal regularity with suitable nonlinear estimates, it is shown that the system admits a unique strong solution locally in time. Secondly, in order to prolong the existence time of the solution, we establish  $H^1L^2 \cap L^2H^2$ -a priori estimates. The key idea is to decompose the horizontal velocity field into the horizontally divergence-free part and the remaining part. We also mention long time behavior of the solution.

# The energy equality of the fractional Navier–Stokes equations in the framework of the Lorentz–Besov spaces

Taichi Eguchi  
Waseda University, Japan  
hasegawa-t@akane.waseda.jp

We find a new sufficient condition to establish the energy equality of the 3D fractional Navier–Stokes equations with the viscous term  $\nu(-\Delta)^\alpha u$ ,  $0 \leq \alpha$ ,  $0 < \nu$  in the whole space. In the case of the Navier–Stokes equations ( $\alpha = 1$ ), Cheskidov–Luo (2020) obtained a sufficient condition for the validity of its energy equality in the framework of the Lorentz–Besov spaces. In this talk, considering a class of the Lorentz–Besov spaces corresponding to that of Cheskidov–Luo, we obtain a new sufficient condition for the energy equality of the fractional Navier–Stokes equations for  $0 \leq \alpha$ . Note that we may regard our result as a generalization of Cheskidov–Luo.